Localization of Acoustic Beacons using Iterative Null Beamforming over Ad-hoc Wireless Sensor Networks

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***Abstract*—In this paper an iterative method to localize and separate multiple audio beacons using the principles of null beam forming is proposed. In contrast to standard methods, the source separation is done optimally by putting a null on all the other sources while obtaining an estimate of a particular source. Also, this method is not constrained by fixed sensor geometry as is the case with general beamforming methods. The wireless sensor nodes can therefore be deployed in any random geometry as required. Experiments are performed to estimate the location and also the power spectral density of the separated sources. The experimental results indicate that the method can be used in ad-hoc, flexible and low-cost wireless sensor network deployment.**

I. Introduction

Source localization is a central problem in wireless sensor networks and has applications in event tracking, multimedia applications and location based services (LBS). Localization and source separation have been extensively dealt with in the context of sensor arrays on a constrained geometry but these methods cannot be directly applied to a random geometry. An energy-based method which estimates distances based on RSS measurements and energy attenuation is proposed in [1]. A Time of Arrival(TOA) based technique which localizes the source directly from TOA estimates is suggested in [2], but it requires particular source signal features to be present. In [3] the problem of a single sound source localization in a distributed sensor environment is addressed. The method, however, only considers the case when there is only one active source at each moment in time. Beamforming and localization on a randomly distributed sensor array system is discussed in [4]. However, the method is limited to the strongest source and the beamforming is not optimal. The beamformer does not put a null on the undesired sources. In [5] the authors suggested a method for source localization using iterative beamforming.

In this work, we propose a method for multiple source separation and localization on an unconstrained geometry. As the method does not rely on planar wave fronts being incident on the sensors, the sources of interest can be both far field and near field. A major advantage of wireless sensor networks against wired sensor networks is the flexibility and low cost of deployment as there is no restriction on the placement of the sensors. The benefit of deploying over a large area is the improvement of signal to noise ratio for some sources which might otherwise have been far off. The work is suitable to be applied to such networks.

Optimal null beamforming maintains a unity gain for the desired source while cancelling all other sources. Hence a better estimate for the source signals can be obtained. We suggest an iterative method for separating the source signals which does an eigenvalue decomposition on the correlation matrix of the signals received at the sensors to successively separate the sources in ascending order of their output powers. Once the sources have been separated, we apply the maximum power collecting principle and find the complex beamforming weights which add the source signal constructively. The time difference of arrival (TDOA) for each of the sources are obtained as the arguments of these beamforming weights. The sources are subsequently localized using maximum likelihood estimation. The rest of the paper is organized as follows. Section 2 defines the problem. Section 3 describes the iterative algorithm used to separate the sources. Section 4 describes the procedure for calculating the TDOAs and localizing the sources. Results are presented in Section 5. We conclude in Section 6.

II. Problem Formulation

Consider a scenario with *N* sources and *M* sensors randomly deployed in the environment. The sensor outputs are generated by the following system model.

$$y_j = \sum_{i=1}^{N} s_i(t - t_{ij}) + v_j \tag{1}$$

where y_j denotes the sensor output of the j^{th} sensor, v_j is the sensor noise, s_i is the source signal of the i^{th} strongest source, t_{ij} are the delays which are calculated as-

$$t_{ij} = ||\mathbf{r_i}^s - \mathbf{r_j}||/c \tag{2}$$

$\mathbf{r_i}^s$ and $\mathbf{r_j}$ being the position vectors of the i^{th} source and j^{th} sensor respectively and c being the speed of propagation of the signal.

The short-time Fourier transform of the input signals y_j is computed to get $y_j(l,k)$. The received signals are stacked up to form a single array $y(l,k)$. The correlation matrix of the sensor output is given by

$$y(l,k) = [y_1(l,k)\ y_2(l,k)\ ...\ y_M(l,k)]^T \qquad (3)$$

$$R(l,k) = E\{y(l,k)y(l,k)^H\} \qquad (4)$$ [1]

Let w_j denote the complex beamforming weight corresponding to the j^{th} sensor. The beamformer output $d(l,k)$ is given by-

$$d(l,k) = \sum_{j=1}^{M} w_j^*(l,k) * y_j(l,k) \qquad (5)$$ [1]

The problem now is to find the signal s_i for each source, $(t_{ij} - t_{ij_{ref}})$ for all sensors with respect to some reference sensor j_{ref} and hence the source location $\mathbf{r_i^s}$. Also source separation proposed herein does not require knowledge of sensor locations but the location of a minimum of three sensors needs to be known for localization.

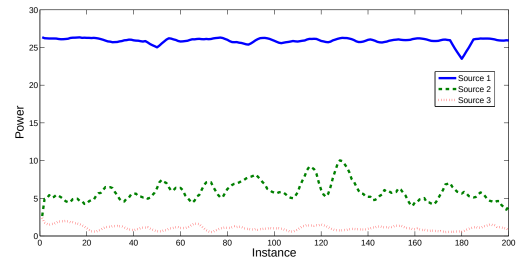
III. Multiple Source Separation using Iterative Null Beamforming

In this section the method used for optimal separation of the source signals is outlined. The method first estimates the weakest source using eigenvalue decomposition of the correlation matrix $R(l,k)$. Once the weakest source(in terms of received power) has been estimated, it is cancelled from the sensor outputs followed by beamforming to obtain the next higher source. This process is repeated to obtain all the sources. The time complexity for separating all sources is $O(NM^3)$.

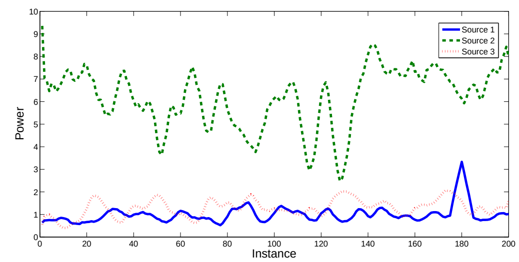
A. Beamforming using Eigenvectors

The principle used for source separation is outlined first. The eigenvector corresponding to the n^{th} highest eigenvalue, referred to as the n^{th} highest eigenvector, nulls out the $(n-1)$ strongest sources. The simulation results shown in Fig. 1 justify this. We have used 3 narrowband sources, 20 sensors and 200 test runs in the simulation setup(described in Section 5). The input powers of the 3 sources are in the ratio $25 : 9 : 1$. For optimal source separation, an estimate for the weakest source is first obtained. We start from the weakest source because the 1st eigenvector does not null out the weaker sources but the eigenvector corresponding to the weakest source nulls out the stronger sources as seen from Fig. 1.

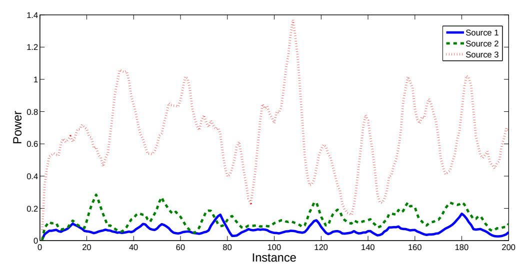
[1] X^H denotes Hermitian of X, X^T denotes transpose of X and x^* denotes complex conjugate of x



(a) Estimated power of the 3 sources after beamforming using first eigenvector.



(b) Estimated power of the 3 sources after beamforming using second eigenvector.



(c) Estimated power of the 3 sources after beamforming using third eigenvector.

Fig. 1. Estimated power of the 3 sources using the 3 eigenvectors for beamforming.

B. Decomposition of Signal Correlation Matrix

The number of sources present should be known before proceeding with source separation. A method for estimating the number of sources is indicated in [6]. The number of eigenvalues of the correlation matrix $R(l,k)$ above a certain threshold gives an estimate of the number of sources present. The eigenvector corresponding to the n^{th} highest eigenvalue, referred to as the n^{th} highest eigenvector, nulls out the $(n-1)$ strongest sources. This idea is first used to obtain an estimate for the weakest source. The N^{th} strongest source s_N is obtained by beamforming using the N^{th} highest eigenvector as the beamforming weights. As the $(N-1)$ strongest sources have been nulled, the estimate d_N for the N^{th} strongest source is optimal.

C. Iterative nulling of undesired sources

To obtain optimal estimates of the sources, all sources other than the required source s_n must be cancelled. The $(n-1)$ sources stronger than the desired source can be removed by using the n^{th} highest eigenvector as the beamforming weights.

The $(N-n)$ weaker sources are removed by cancelling them out, as optimal estimates have already been obtained for them. Hence after the weakest source s_N has been estimated it is cancelled from the sensor outputs. To do this, the time delay t_{Nj} between the estimated d_N and the signal from the source N arriving at the sensor j should be known. Generalized Cross Correlation(GCC) method is used to find the time delays [7]. The idea behind this method is that the time delay between two time shifted signals can be found by computing the cross-correlation function of the two signals. The lag at which the cross-correlation function has its maximum is taken as the time delay between the two signals. Subsequently, the signal d_N is shifted by t_{Nj} and subtracted from the sensor output $y_{j,N}$ to obtain a new sensor output $y_{j,N-1}$.

D. Reconstruction of multiple sources using iterative beamforming

The $N-1$ strongest source s_{N-1} can now be obtained by using the $N-1$ highest eigenvector as the beamforming weights on the new sensor outputs $y_{j,N-1}$. This process is repeated iteratively to obtain all the sources s_i. At each iteration n starting from 1, we cancel the estimate of the previously determined signal s_{N-n+2} from the sensor outputs using cross correlation method and then obtain s_{N-n+1} using beamforming with the $(N-n+1)^{th}$ eigenvector as the beamforming weights on the new sensor outputs $y_{j,N-n+1}$.

Algorithm 1 Multiple Source Separation using Iterative Null Beamforming.

1: Initialize $n \leftarrow N$, $y_N(l,k) \leftarrow y(l,k)$
2: Compute the correlation matrix $R(l,k) = E\{y_n(l,k)y_n(l,k)^H\}$ and compute the n^{th} highest eigenvector $V_n(l,k)$
3: Set $w_n(l,k) \leftarrow V_n(l,k)$. $d_n(l,k) = \sum_{j=1}^{M} w_j^*(l,k) * y_j(l,k)/\sqrt{M}$ gives the n_{th} strongest source.
4: Compute the delays $t_{nj} = \frac{1}{F_s} max_i \sum_{k=0}^{W-1} \frac{1}{W}(y_{j,n}(l,k)d_n^*(l,k))e^{j\frac{2\pi ik}{W}}$, where W is the length of the STFT window and F_s is the sampling frequency.
5: Obtain $d_{j,n}(l,k)$ by shifting $d_n(l,k)$ by t_{nj}.
6: Set $y_{j,n-1}(l,k) \leftarrow (y_{j,n}(l,k) - d_{j,n})$, $n \leftarrow n-1$
7: Return to step 2, till $n = 0$

IV. Multiple Source Localization using Maximum Power Collection Principle

Once the sources have been estimated the TDOA estimates are computed. The sources are then localized using maximum likelihood principle. The TDOA estimates are obtained by applying maximum power collection principle. The time complexity for localizing all sources is $O(N\tilde{M}^3)$, where $\tilde{M}$ is the number of sensors whose location is known.

A. Maximum Power Collection Principle[4]

If the beamforming weights $w(l,k)$ are chosen to maximize the output power, subject to unity gain i.e.,

$$w(l,k) = \max(w(l,k)(w(l,k)^H R(l,k)w(l,k)) \quad \text{such that } ||w(l,k)|| = 1 \tag{6}$$

The desired $w(l,k)$ is then given by the eigenvector corresponding to largest eigenvalue of correlation matrix, $R(l,k)$. The weights which maximize the output power would also be those which sum the strongest signal constructively between the sensors and maximize it's power. Hence the complex beamforming weights which add strongest signal constructively can be obtained.

B. Obtaining the TDOA estimates

To add signals at different microphones constructively, these must be aligned together. The phases by which the different sensor signals are shifted to align them are used to calculate the TDOA estimates for that source. The arguments of the complex beamforming weights used to maximize the power of a source gives the phase shift applied to the sensor signals to maximize the power of that source. However, accurate results for TDOA can only be obtained when the dominant eigenvector of the correlation matrix is used to maximize power for the strongest source. Using the next higher eigenvectors to localize the next higher sources can give inaccurate estimates. Hence the source whose TDOA estimates are to be determined must be the strongest source. This is taken care of by cancelling all sources having higher power than the source whose TDOA estimates are required from the sensor outputs. As optimal estimates $d_n(l,k)$ for all the sources have already been determined, the sources having higher powers are cancelled by subtracting the estimated source from each of the input signals $y_j(l,k)$. Additionally, $d_n(l,k)$ has to be appropriately shifted to $d_{j,n}^{'}(l,k)$ as done earlier by finding the lag which maximizes the cross correlation function of $y_j(l,k)$ and $d_n(l,k)$.

$$y'_{j,n}(l,k) = y_j(l,k) - \sum_{t=1}^{n-1} d_{j,t}^{'}(l,k) \tag{7}$$

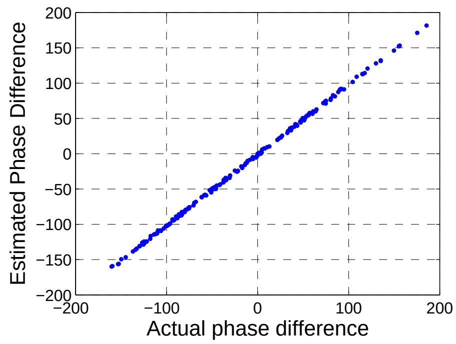
Subsequently, the dominant eigenvector V_n of the correlation matrix $R'_n(l,k)$ of the sensor signals $y'_{j,n}(l,k)$ is used to find the TDOA estimates for the n^{th} source using Eq. 8.

$$w_k \tau_{nj}(l,k) = arg(V_n(j)) - arg(V_n(j_{ref})) \tag{8}$$

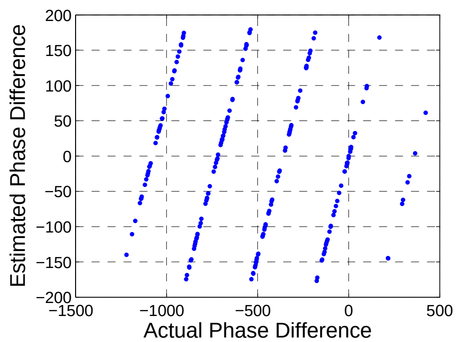
w_k denotes the frequency associated with frequency bin k.

C. Effect of Spatial Aliasing on TDOA estimation

When obtaining the TDOA estimates, spatial aliasing is observed when the source frequency is high i.e. when $\tau_{ij}f_i > 1$ the phase difference obtained differs from the actual phase difference by a factor of $2n\pi$. This is illustrated in Fig. IV-C. Each point in these figures denotes the actual and determined phase differences between a sensor and the reference sensor. In



(a) Source frequency=50 Hz. The TDOAs are correctly determined.



(b) Source frequency=50 kHz. Spatial aliasing is observed.

Fig. 2. Figure illustrating the estimated vs actual TDOA estimates under spatial aliasing

Fig. 2(a) the phase differences are correctly obtained. Multiple parallel lines with slope of 1 are obtained in Fig. 2(b) which correspond to the phase differences of $2n\pi$ as discussed. Some heuristics can be used to deal with spatial aliasing. The sensor pairs which have a large inter sensor distance or are known to be far away from the source are likely to have a larger value of τ_{ij} and can be omitted from the calculation if the location of multiple sensors is known.

D. Localization using TDOA estimates

For localization of the sources, an efficient hyperbolic location estimator [8] is used. The estimator is an approximate realization of the maximum likelihood estimator and attains the Cramér-Rao lower bound for small errors. The method is non-iterative and gives an explicit, closed form solution. If the location of three sensors in the wireless sensor network is known, the source location is given by the intersection of the hyperbolic curves.

$$\begin{bmatrix} x_i \\ y_i \end{bmatrix} = -\begin{bmatrix} x_{2,1} & y_{2,1} \\ x_{3,1} & y_{3,1} \end{bmatrix}^{-1} \times \left\{ \begin{bmatrix} d_{21,i} \\ d_{31,i} \end{bmatrix} d_{1,i} + \frac{1}{2} \begin{bmatrix} d_{21,i}^2 - ||\mathbf{r_2}|| + ||\mathbf{r_1}|| \\ d_{31,i}^2 - ||\mathbf{r_3}|| + ||\mathbf{r_1}|| \end{bmatrix} \right\} \quad (9)$$

Here x_i denotes x coordinate of Source i, $x_{2,1}$ denotes x coordinate of $\mathbf{r_2} - \mathbf{r_1}$ and so on. $d_{1,i} = ||\mathbf{r_i}^s - \mathbf{r_1}||$ and $d_{21,i} = d_{2,i} - d_{1,i}$. The knowledge of the location of additional sensors lets us use a two step Least Squares (LS) method which is an approximation of the ML estimator. However, the wireless sensor nodes are randomly deployed so the location of all the nodes may not be known. Hence, the flexibility of localization using a minimum of three sensors is desirable.

E. Time Synchronization for Localization

As all the sensor data is sent to the same computation centre, the sensor nodes need to be time synchronized for localization. Synchronization is done prior to source separation by implementing Reference Broadcast Synchronization Method, described in [9]. Time synchronization can be avoided if the source location of sufficient number of nodes are available as the hyperbolic location estimator can give a ML estimate of the source location.

V. Experiments on Multiple Acoustic Beacon Localization

In this section experimental results obtained by using the proposed method for source separation and localization on multiple acoustic beacons is described.

A. Simulations

We have used $N = 3$ acoustic beacons, referred as S_1, S_2 and S_3, and $M = 20$ sensors. The sensors have been deployed randomly. The acoustic beacons have spectral peaks at $f = 250Hz, 150Hz$ and $50Hz$ respectively. The SNR ratio at the sensor inputs is 15. Room reverberation effects are not taken into account in the current scenario. Fig. 3 illustrates the setup.

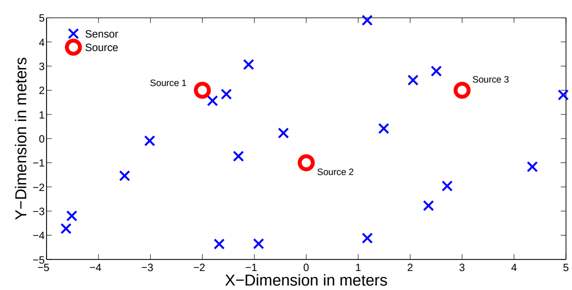


Fig. 3. Experimental setup for multiple acoustic beacon localization with 3 acoustic beacons and 20 sensors.

B. Experimental deployment

An experimental WSN is also deployed using National Instruments NI cDAQ-9191 and PCB 130E20 microphones. The data is sent to the PC over IEEE 802.11 Wi-Fi. Fig. 4 shows the experimental setup. We have used $M = 4$ sensors to localize $N = 2$ audio beacons in the experimental deployment.

C. Results on Multi Source Separation

The results for multi source separation on Fig. 3 using proposed iterative null beamformer are summarized in Table I. The power corresponding to the 3 sources in the estimated signals for each source as a percentage of their powers in the input signal at the reference sensor is shown.

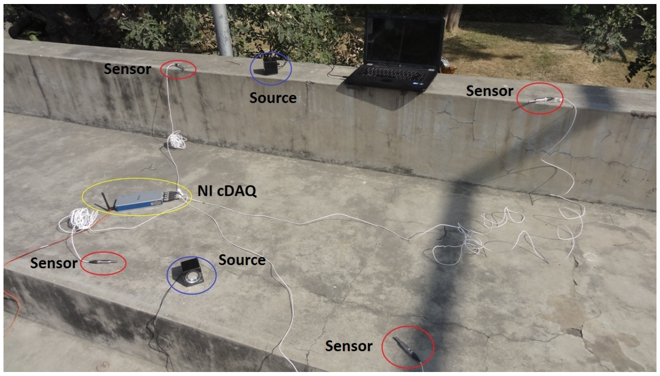


Fig. 4. The experimental setup showing 4 PCB 130E20 microphones connected to NI cDAQ-9191 and 2 audio sources

TABLE I
OUTPUT POWER OF THE ESTIMATED SIGNALS AS A PERCENTAGE OF THEIR POWERS IN THE INPUT SIGNAL

Signal	S_1	S_2	S_3
Estimated S_1	98%	$1.5 * 10^{-5}\%$	$2.0 * 10^{-3}\%$
Estimated S_2	$8.2 * 10^{-6}\%$	96%	$1.2 * 10^{-3}\%$
Estimated S_3	$6.6 * 10^{-5}\%$	$3.3 * 10^{-3}\%$	95%

D. Results on Multi Source Localization

For testing the localization algorithm, 1000 tests were run as per setup shown in Fig. 3. In each test, different locations of the sensors and sources were taken within the region of interest, $(-5m, 5m) \times (-5m, 5m)$. Fig. 5 shows the radial error deviation plots of the obtained location. The radial error is taken as the distance from the actual location. The figure is shown on a scale of 0-2m whereas the actual dimension of the setup is 10m. As the area ΔA corresponding to given radial error distance r is $\propto r$, the plots have been appropriately scaled by the radial distance. The experimental setup shown in Fig. 4 gives similar results with the source getting localized to within 10% error in 90% of the test runs. The Cramer-Rao Lower Bound(CRLB) plot for the experimental results is presented in Fig. 6. The minimum variance obtained herein is 5.8143m.

VI. CONCLUSION

In this paper an algorithm to separate and localize multiple sources over sensor nodes that can follow any random geometry is proposed. The additional novelty of the proposed approach lies in the enforcement of the null in the direction of the undesired sources in an iterative fashion. The experimental results on acoustic beacon localization illustrate that the beacons can be localized with a high degree of reliability.

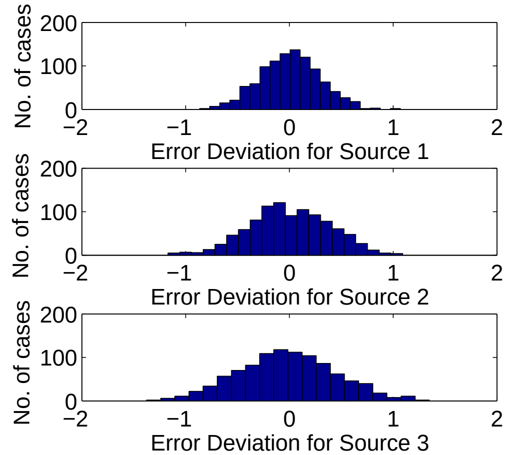


Fig. 5. Radial error deviation of the source locations

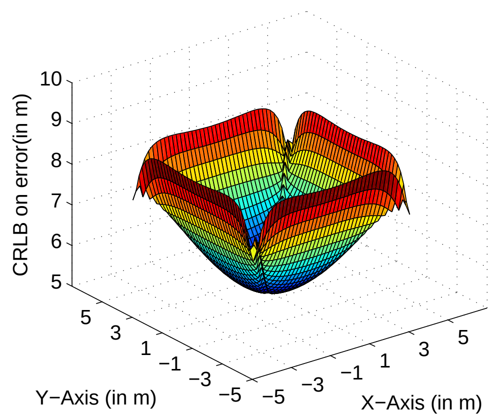


Fig. 6. CRLB Analysis

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