Probability Exercises

CSCI 567 Machine Learning

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Instructor: Vatsal Sharan

MULTIPLE-CHOICE QUESTIONS: one or more correct choices for each question.

Q1 A bag contains 2 red balls and 3 blue balls. First, Alice draws a ball from the bag randomly (and removes it from the bag). Then, Bob draws a ball randomly too. 1) What is the probability that Alice gets a red ball and Bob gets a blue ball? 2) What is the probability that Alice gets a blue ball given that Bob gets a blue ball?

(a) $\frac{3}{10}$ and $\frac{1}{2}$

(b) $\frac{3}{10}$ and $\frac{2}{5}$

(c) $\frac{6}{25}$ and $\frac{1}{2}$

(d) $\frac{6}{25}$ and $\frac{2}{5}$

Q2 For events A, B and C, which of the following identities are correct?

(a) $P(A) - P(A \cap B) = P(A \cup B) - P(B)$

(b) $P(A \cup B) \leq P(A) + P(B) - P(A)P(B)$

(c) $P(A) = P(A \cap C) + P(A \cap \overline{C})$, where $\overline{C}$ denotes the complement of event C.

(d) $P(A) = P(A|C) + P(A|\overline{C})$, where $\overline{C}$ denotes the complement of event C.

Q3 For events A, B, C and $Z_1, \ldots, Z_T$, which of the following identities are correct?

(a) $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$

(b) $\frac{P(A|B,C)}{P(A|C)} = \frac{P(B|A,C)}{P(B|C)}$

(c) $P(\bigcap_{t=1}^{T} Z_t) = \Pi_{t=1}^{T} P(Z_t)$

(d) $P(\bigcap_{t=1}^{T} Z_t) = \Pi_{t=1}^{T} P(Z_t|Z_1, \ldots, Z_{t-1})$

Q4 Which of the following statements on the density function of a Gaussian distribution are true?

(a) The density for a one-dimensional Gaussian distribution with mean μ and variance σ^2 is $f(x) \propto \exp\left(\frac{(x-\mu)^2}{2\sigma^2}\right)$.

(b) The density for a one-dimensional Gaussian distribution with mean μ and variance σ^2 is $f(x) \propto \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$.

(c) The density for a d-dimensional Gaussian distribution with mean $\mu \in \mathbb{R}^d$ and covariance matrix $\Sigma \in \mathbb{R}^{d \times d}$ is $f(\mathbf{x}) \propto \exp\left(-\frac{1}{2}(\mathbf{x} - \mu)^\top \Sigma (\mathbf{x} - \mu)\right)$.

(d) The density for a d-dimensional Gaussian distribution with mean $\mu \in \mathbb{R}^d$ and covariance matrix $\Sigma \in \mathbb{R}^{d \times d}$ is $f(\mathbf{x}) \propto \exp\left(-\frac{1}{2}(\mathbf{x} - \mu)^\top \Sigma^{-1} (\mathbf{x} - \mu)\right)$.

Q5 Which of the following statements are true?

(a) Suppose X and Y are two jointly Gaussian random variables. Then $Z = X - 2Y$ is also Gaussian.

(b) Suppose X and Y are two jointly Gaussian random variables. Then the marginal distribution of X is also Gaussian.

(c) Suppose X and Y are two jointly Gaussian random variables. Then the conditional distribution of X given Y is also Gaussian.

(d) For a random vector $X \in \mathbb{R}^n$, its covariance matrix is $\mathbb{E}[XX^\top] - \mathbb{E}[X]\mathbb{E}[X]^\top$.

SHORT-ANSWER QUESTION.

Q6 Suppose your spam classification software gives the guarantee that (1) if an email is spam, it will mark it as spam with probability 90%, (1) if an email is not spam, it will only mark it as spam with probability 10%. Suppose you know that 1% of all your emails are spam. If your spam classification software marks a certain email as spam, what is the probability that it is actually spam?